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Aharonov–Bohm–Casher oscillations in strongly correlated electron systems at finite temperature

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Abstract. Persistent charge and spin currents due to Aharonov–Bohm and Aharonov–Casher interferences of correlated electrons moving along a mesoscopic ring are discussed at finite temperature. As a function of applied flux the ground-state persistent currents have the shape of a generalized saw-tooth, i.e. they consist of piecewise straight segments. The periods and amplitudes of the oscillations are associated with the properties of the Fermi surface of the elementary excitations (two Dirac seas), namely the group velocities and the matrix of dressed generalized charges (Luttinger parameters). The temperature reduces the amplitudes of oscillation by smearing the Fermi surface in a similar way to that for the de Haas–van Alphen effect in 3D metals. The amplitude of higher harmonics decreases more quickly with T than the fundamental one, changing the saw-tooth to a more sinusoidal form with much smaller amplitude. The controlling parameters are the ratios of the thermal energy to the level spacings in the ring. The results are discussed in the context of the exact Bethe *ansatz* solutions for the Hubbard chain and the supersymmetric t–J model.

1. Introduction

The exact solution of numerous one-dimensional models by means of Bethe's *ansatz* and the quantum inverse scattering method in conjunction with field-theoretical treatments has provided deep insight into the ground-state properties, classification of states, thermodynamics, and the asymptotic behaviour of correlation functions (see reference [1] for an extensive review).

The finite length of a conducting or magnetic ring can manifest itself in several ways. (i) The contribution of impurities to extensive quantities (e.g. the energy, susceptibility, specific heat) can become large and observable in mesoscopic systems, like for instance in the case of the Kondo effect. (ii) The finite length of a ring with periodic boundary conditions gives rise to quantum topological effects, i.e. persistent currents with oscillation periods given by interference patterns of the Aharonov–Bohm (AB) [2] and Aharonov–Casher (AC) [3] type due to the finite spacing of the energy levels. (iii) The finite-size corrections to the ground-state energy determine the low-energy excitation spectrum and via conformal field theory the critical exponents of the asymptotic long-distance dependence of correlation functions [4–7].

In the AB effect [2] the wavefunction of charged particles moving along a ring picks up a phase proportional to the magnetic field flux threading the ring. Dual to the AB effect is the AC effect [3] in which the wavefunction of particles with a magnetic moment acquires a phase due to a radial electric field caused for instance by a straight homogeneously charged line enclosed by the ring. The quantization of the flux or the phase (modulo 2π) leads to periodic oscillations of the current [8]. Persistent currents have been observed experimentally in small metal and semiconductor rings [9].

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Persistent currents in interacting systems were studied theoretically within the framework of the exact Bethe *ansatz* method [10–20] and the bosonization technique [21, 22]. The system responds to a magnetic or electric flux by virtually creating and annihilating states at the left and right Fermi points of the Dirac sea conserving the total charge and spin. The change of the energy with the flux is on a mesoscopic scale, i.e. proportional to L^{-1} for bands that are gapless in the thermodynamic limit, where L is the length of the ring. The low-energy excitation spectrum is given by the conformal towers of the system. In the ground state the persistent current oscillations have the shape of a generalized saw-tooth. A finite temperature and impurity scattering of the electrons smears the Fermi surface and strongly decreases the amplitude of the higher-harmonic content. The experimentally observed oscillations are then nearly sinusoidal. In addition, the coupling of the flux to the particles may give rise to more than just one period of oscillation. AB oscillations at finite T have been studied previously for *spinless* fermions via the bosonization and Bethe *ansatz* methods [21, 23].

In this paper we present a theoretical study of the AB and AC oscillations of strongly interacting electrons moving along a ring at finite temperatures. We consider the cases of two integrable systems, namely the Hubbard and supersymmetric t–J models. The structure of the Bethe *ansatz* solution is different for these two models, and they are representatives of the two most common generic situations. The results are then believed to be valid in general for Luttinger liquids and are not specific to the integrability. In section 2 we state the conformal towers (finite-size corrections to the ground-state energy) for a general integrable model and the expression for the persistent current at $T = 0$. The formulation for finite temperatures is introduced in section 3. The oscillations are strongly suppressed with temperature due to the incoherence introduced by the smearing of the Fermi surface with T. In section 4 we specifically address the situations of the Hubbard and supersymmetric t–J models. Concluding remarks are presented in section 5.

2. The finite-size excitation spectrum

The AB and AC effects are mesoscopic oscillations of the ground-state energy as a function of a magnetic or electric field flux. The amplitude of these oscillations is proportional to the level spacing in the ring, i.e. $\hbar v_F/L$, where L is the size of the ring and v_F is the Fermi velocity. The persistent current oscillations arise from virtual transitions of low-lying excitations from one Fermi point to the other, induced by a nonzero quasimomentum due to the field flux. The AB and AC oscillations are then determined by the mesoscopic (finite-size) corrections to the energy.

Expanding the ground-state energy for an interacting Fermi gas as a power series in L^{-1} we have [24]

$$E(L) = LE_\infty + L^{-1}E_{mes} + \cdots \tag{2.1}$$

where E_∞ is the ground-state energy density and E_{mes} is the mesoscopic contribution. We consider here a single-electron-band model, like for instance the Hubbard or t–J models. In view of the charge and spin separation in one dimension, charges and spins propagate independently. The wavenumbers of charges (spinons) have all to be different and hence the states follow Fermi statistics. The energies of charge and spin states are given in terms of two energy bands, one for the charges and one for the spin states (labelled here with 1 and 2), each with a Fermi surface that depends on the chemical potential and the magnetic

field. Both bands contribute to the mesoscopic corrections [24]

$$E_{mes} = \sum_{i=1,2}(2\pi v_i)\left\{\left[\frac{1}{2}\sum_{j=1,2}\hat{z}_{ij}^{-1}\,\Delta N_j\right]^2 + n_i^+ + n_i^- - \frac{1}{12}\right\} + \sum_{i=1,2}(2\pi v_i)\left[\sum_{j=1,2} z_{ji}\big(D_j + \{\vartheta_j\}\big)\right]^2 \tag{2.2}$$

where v_1 and v_2 denote the group velocities of the two bands (the Fermi velocities within the charge and spin sectors are in general different), and $\hat{z}$ is the 2×2 matrix of generalized dressed charges describing the coupling of the two Fermi surfaces. The quantum numbers ΔN_1 and ΔN_2 are the departures in the populations of the bands from their average values, D_1 and D_2 correspond to the backward-scattering quantum numbers within each band, and the $n_i^{\pm}$ represent the quantum numbers of the particle–hole-like excitations about the four Fermi points. Finally, the $\{\vartheta_j\}$ are the fractional part (to the closest integer) of the AB and AC phase shifts due to the coupling of the charges (spins) to the magnetic (electric) flux. Equation (2.2) is the general form of the finite-size corrections to the ground-state energy (long-wavelength low-energy excitations) of a two-component Luttinger liquid (two Gaussian conformal field theories of central charge one) and holds independently of the integrability of the underlying model.

The Bethe *ansatz* solution of strongly correlated electron models provides the possibility of actually calculating the Luttinger liquid parameters v_j and z_{ij}. The v_i also determine the spin and charge susceptibilities and the γ-coefficient of the specific heat. The integral equations satisfied by the excitations and the dressed charges are addressed in section 4 for the Hubbard and the supersymmetric t–J models. For the purpose of discussing the temperature dependence of the persistent currents we only need the general expression (2.2), but not the actual values of the Luttinger parameters.

From equation (2.2), the flux-dependent part of the energy is [10, 16–18]

$$\Delta E(\vartheta_1,\vartheta_2) = \frac{2\pi}{L}v_1\Big[z_{11}(D_1+\{\vartheta_1\}) + z_{21}(D_2+\{\vartheta_2\})\Big]^2 + \frac{2\pi}{L}v_2\Big[z_{12}(D_1+\{\vartheta_1\}) + z_{22}(D_2+\{\vartheta_2\})\Big]^2 \tag{2.3}$$

which contains interference terms of the two Dirac seas of the type $(D_1+\{\vartheta_1\})(D_2+\{\vartheta_2\})$. For the finite-temperature analysis it is therefore more convenient to study directly the persistent currents, defined as $j_i = -\partial\,\Delta E/\partial\vartheta_i$ for $i = 1, 2$. The currents are linear in $(D_i + \{\vartheta_i\})$ and can be written as

$$\begin{aligned} j_i(\vartheta_1,\vartheta_2) &= j_i^{(1)}(\vartheta_1,\vartheta_2) + j_i^{(2)}(\vartheta_1,\vartheta_2) \\ j_i^{(l)}(\vartheta_1,\vartheta_2) &= -\frac{4\pi}{L}v_l z_{il}\Big[z_{1l}(D_1+\{\vartheta_1\}) + z_{2l}(D_2+\{\vartheta_2\})\Big]. \end{aligned} \tag{2.4}$$

For the ground state, $j_i(\vartheta_1,\vartheta_2)$ as a function of the fluxes consists of piecewise straight segments, i.e. a generalized saw-tooth, with in general two periodicities. In section 4 we discuss the oscillations for the Hubbard and supersymmetric t–J models.

3. Temperature dependence of the oscillations

To calculate the finite-T effects on the persistent current we first consider the change in the free energy due to the temperature:

$$F(T;\vartheta_1,\vartheta_2) = E_G(\vartheta_1,\vartheta_2) + \Delta F(T;\vartheta_1,\vartheta_2) \tag{3.1}$$

where E_G is the ground-state energy of the finite-size system, and ΔF can be expressed as

$$\Delta F(T;\vartheta_1,\vartheta_2) = -\frac{TL}{2\pi}\sum_{i=1,2}\int \mathrm{d}p\ \ln\bigl[1+\exp(-\varepsilon_i(p;\vartheta_1,\vartheta_2)/T)\bigr] \tag{3.2}$$

where $\varepsilon_i(p)$, $i=1,2$, are the energies of the excitations due to backward scattering (virtual transitions from one Fermi point to the other) in the two bands. Expression (3.2) follows from the Fermi statistics of the charge and spin states and the finite-size corrections (2.3), and p is the momentum transfer across the Fermi surfaces. Since T and L^{-1} are variables of the same order, the contribution of ΔF is already mesoscopic and does not require a finite-size expansion. Due to the one dimensionality of the interacting electron system, the thermal population of the excitations is governed by the Fermi statistics. Note that $\Delta F(T)$ vanishes as $T\to 0$. The expression for $\Delta F(T)$ is valid only for low temperatures, since we neglected excitation branches without a Fermi surface. Hence, at finite temperature the current consists of a $T=0$ contribution (saw-tooth-like) and one that is temperature dependent. From (3.2) the latter for low T is given by

$$\Delta j_i^{(l)}(T;\vartheta_1,\vartheta_2) = -\frac{L}{2\pi}\int \mathrm{d}p\ \frac{\partial\varepsilon_l(p;\vartheta_1,\vartheta_2)/\partial\vartheta_i}{1+\exp(\varepsilon_i(p;\vartheta_1,\vartheta_2)/T)}. \tag{3.3}$$

Keeping the number of particles in each band constant and linearizing the dispersions about the Fermi points, the backward-scattering excitation energies are given by

$$\varepsilon_l(p;\vartheta_1,\vartheta_2) = \frac{2\pi}{L}v_l\bigl|z_{1l}(D_1+\{\vartheta_1\}) + z_{2l}(D_2+\{\vartheta_2\})\bigr| \tag{3.4}$$

where the D_i, depending on the initial conditions, are either integers or half-integers. Note that the ε_l represent excitation energies and are always positive. It follows that

$$\begin{aligned}\frac{\partial\varepsilon_l}{\partial\vartheta_i} &= \frac{2\pi}{L}v_l z_{il}\,\mathrm{sgn}\Bigl(z_{1l}(D_1+\{\vartheta_1\}) + z_{2l}(D_2+\{\vartheta_2\})\Bigr)\\ \mathrm{d}p_l &= \frac{2\pi}{L}[z_{1l}\,\Delta D_1 + z_{2l}\,\Delta D_2]\end{aligned} \tag{3.5}$$

where ΔD_1 and ΔD_2 are integers. The integration in equation (3.3) is now reduced to a sum over the integers ΔD_1 and ΔD_2, so

$$\Delta j_i^{(l)}(T;\vartheta_1,\vartheta_2) = -2v_l z_{il}\frac{2\pi}{L}\sum_{m=1,2} z_{ml}\sum_{D_m}\frac{\mathrm{sgn}(D_m+\{\vartheta_m\})}{1+\exp\bigl[(2\pi v_l/TL)z_{ml}|D_m+\{\vartheta_m\}|\bigr]}. \tag{3.6}$$

The first factor of 2 arises because each band has two Fermi points that contribute equally. It is of course the same to sum over D_m or ΔD_m, and while the sum over one quantum number is carried out, the other one is kept at the Fermi level. The temperature only enters the exponential via a dimensionless parameter

$$2\pi v_l z_{ml}/LT$$

i.e. the level spacing over the temperature, which controls the smearing of the Fermi surfaces and hence determines the amplitudes of oscillation. Since only the states close to the Fermi surface contribute because of the exponential, we can extend the sum over D_m from $-\infty$ to ∞. If the D_m are half-integers, then to simplify the notation, we can absorb $1/2$ into $\{\vartheta_m\}$ and reintroduce the parity at a later stage. Hence, at this point we will consider the D_m to be integers.

It is convenient to rewrite equation (3.6) using Poisson's formula; using the notation

$$x = (2\pi/L)D_m$$

we obtain

$$\Delta j_i^{(l)}(T;\vartheta_1,\vartheta_2) = -2v_l z_{il}\sum_{m=1,2} z_{ml}\int_{-\infty}^{\infty} \mathrm{d}x\ \frac{\operatorname{sgn}(x+(2\pi/L)\{\vartheta_m\})}{1+\exp\left[(v_l z_{ml}/T)|x+(2\pi/L)\{\vartheta_m\}|\right]}$$
$$-\,4v_l z_{il}\sum_{m=1,2} z_{ml}\sum_{s=1}^{\infty}(-1)^s\int_{-\infty}^{\infty}\mathrm{d}x\ \frac{\cos(Lsx)\operatorname{sgn}(x+(2\pi/L)\{\vartheta_m\})}{1+\exp\left[(v_l z_{ml}/T)|x+(2\pi/L)\{\vartheta_m\}|\right]}. \tag{3.7}$$

The non-oscillating terms in (3.7) vanish by symmetry of the integrand. The oscillatory terms can be reduced to

$$\Delta j_i^{(l)}(T;\vartheta_1,\vartheta_2) = -4v_l z_{il}\sum_{m=1,2} z_{ml}\sum_{s=1}^{\infty}(-1)^s\sin\bigl(2\pi s\{\vartheta_m\}\bigr)\int_{-\infty}^{\infty}\mathrm{d}x\ \frac{\sin(Lsx)\operatorname{sgn}(x)}{1+\exp\left[v_l z_{ml}|x|/T\right]} \tag{3.8}$$

which using a partial integration yields

$$\Delta j_i^{(l)}(T;\vartheta_1,\vartheta_2) = -\frac{4v_l z_{il}}{L}\sum_{m=1,2} z_{ml}\sum_{s=1}^{\infty}\frac{(-1)^s}{s}\sin\bigl(2\pi s\{\vartheta_m\}\bigr)$$
$$+\,\frac{v_l^2 z_{il}}{TL}\sum_{m=1,2} z_{ml}^2\sum_{s=1}^{\infty}\frac{(-1)^s}{s}\sin\bigl(2\pi s\{\vartheta_m\}\bigr)\int_{-\infty}^{\infty}\mathrm{d}x\ \frac{\cos(Lsx)}{\cosh^2\bigl(v_l z_{ml}x/2T\bigr)}. \tag{3.9}$$

Consider now the Fourier-series expansion of the saw-tooth function $\{x\}$, defined as x in the interval $-1/2 < x < 1/2$ and periodically continued with period one:

$$\{x\} = -\frac{1}{\pi}\sum_{s=1}^{\infty}\frac{(-1)^s}{s}\sin(2\pi sx). \tag{3.10}$$

Hence, the first term of expression (3.9) is just

$$\frac{4\pi v_l z_{il}}{L}\sum_{m=1,2} z_{ml}\{\vartheta_m\} \tag{3.11}$$

and exactly cancels the $T = 0$ ground-state current, equation (2.4). Carrying out the x-integral in the second term of (3.9), we finally have that

$$j_i(T;\vartheta_1,\vartheta_2) = j_i^{(1)}(T;\vartheta_1,\vartheta_2) + j_i^{(2)}(T;\vartheta_1,\vartheta_2)$$
$$= 4\pi T\sum_{l,m=1,2} z_{il}\sum_{s=1}^{\infty}(-1)^s\frac{\sin\left[2\pi s(\vartheta_m+D_m)\right]}{\sinh(\pi LsT/v_l z_{ml})} \tag{3.12}$$

where we reintroduced the initial phase due to the quantum numbers D_m (parity effect). With increasing $(LT/v_l z_{ml})$, i.e. temperature over level spacing, the amplitudes of the harmonic content decreases rapidly, changing the saw-tooth pattern into a more sinusoidal one.

4. Application to integrable systems

In this section we discuss the persistent current oscillations at finite temperatures for two integrable models, the Hubbard and the supersymmetric t–J models. The Bethe *ansatz* ground states for these models are different, and hence they have different AB and AC interference patterns. The Hubbard and the supersymmetric t–J models are representatives of the two most commonly found structures of Bethe *ansatz* solutions of integrable correlated electron systems. The results presented here are believed to be generic to all systems with Luttinger liquid properties and are not specific to the condition of integrability.

4.1. The Hubbard model

The Hubbard Hamiltonian is given by

$$H = -\sum_{i\sigma}\left(c^{\dagger}_{i\sigma}c_{i+1\sigma} + c^{\dagger}_{i+1\sigma}c_{i\sigma}\right) + U\sum_{i} n_{i\uparrow}n_{i\downarrow} \tag{4.1}$$

where $c^{\dagger}_{i\sigma}$ creates an electron of spin σ at the site i and $n_{i\sigma} = c^{\dagger}_{i\sigma}c_{i\sigma}$. Here we equated the hopping matrix element to 1. The Hubbard model has been diagonalized within the framework of Bethe's *ansatz* in terms of two sets of quantum numbers, $\{I_n\}$, $n = 1, \ldots, N_c$ for N_c charges and $\{J_\alpha\}$, $\alpha = 1, \ldots, N_s$ for N_s reversed spins [25]. These quantum numbers are integers or half-integers depending on the parities of N_c and N_s, i.e. $I_n = N_s/2$ (mod 1) and $J_\alpha = (N_c + N_s + 1)/2$ (mod 1). All quantum numbers within one set have to be different (Fermi statistics). In the ground state the quantum numbers are densely distributed, i.e. they form a sequence from I_{min} to I_{max} and from J_{min} to J_{max}, respectively, without leaving holes. The backward-scattering quantum numbers D_c and D_s are then determined by $2D_c = I_{min} + I_{max}$ (mod 1) and $2D_s = J_{min} + J_{max}$ (mod 1) [24].

Each state of the system is uniquely determined by a set of quantum numbers $\{I_n\}$ and $\{J_\alpha\}$. The quantum numbers determine the generalized wavenumbers (rapidities) for the propagation of charges and spinons. For the ground state the occupation of states follows from Fermi's statistics and the dressed energy potentials, $\varepsilon(k)$ and $\varphi(\Lambda)$, which satisfy the Fredholm integral equations [26]

$$\begin{aligned}
\varepsilon(k) &= -2\cos k - \mu - H/2 + \int_{-B}^{B} \mathrm{d}\Lambda\ a_1(\Lambda - \sin k)\varphi(\Lambda) \\
\varphi(\Lambda) &= H - \int_{-B}^{B} \mathrm{d}\Lambda'\ a_2(\Lambda - \Lambda')\varphi(\Lambda') + \int_{-Q}^{Q} \mathrm{d}k\ \cos k a_1(\Lambda - \sin k)\varepsilon(k)
\end{aligned} \tag{4.2}$$

where $a_n(\Lambda) = (nU/4\pi)/(\Lambda^2 + (nU/4)^2)$. The integration limits Q and B correspond to the Fermi surfaces of charges and spinons, respectively, and are determined by the zeros of the energies, $\varepsilon(\pm Q) = 0$ and $\varphi(\pm B) = 0$. The distribution densities for the rapidities, $\rho(k)$ and $\sigma(\Lambda)$, satisfy similar integral equations [25]:

$$\begin{aligned}
\rho(k) &= \frac{1}{2\pi} + \cos k \int_{-B}^{B} \mathrm{d}\Lambda\ a_1(\Lambda - \sin k)\sigma(\Lambda) \\
\sigma(\Lambda) &= -\int_{-B}^{B} \mathrm{d}\Lambda'\ a_2(\Lambda - \Lambda')\sigma(\Lambda') + \int_{-Q}^{Q} \mathrm{d}k\ a_1(\Lambda - \sin k)\rho(k).
\end{aligned} \tag{4.3}$$

The elementary charge and spin excitations of the Hubbard chain are given by $\Delta E_c(k) = |\varepsilon(k)|$ and $\Delta E_s(\Lambda) = |\varphi(\Lambda)|$ with respective momenta

$$p_c(k) = 2\pi\int_0^k \mathrm{d}k'\ \rho(k') \qquad p_s(\Lambda) = 2\pi\int_0^{\Lambda} \mathrm{d}\Lambda'\ \sigma(\Lambda').$$

From the definition of Q and B the excitation energies vanish at the respective Fermi points. Linearizing about the Fermi level we obtain the group velocities

$$v_c = \left.\frac{\partial\varepsilon}{\partial k}\right|_Q [2\pi\rho(Q)]^{-1} \qquad v_s = \left.\frac{\partial\varphi}{\partial\Lambda}\right|_B [2\pi\sigma(B)]^{-1}. \tag{4.4}$$

The remaining Luttinger parameters are the matrix of dressed generalized charges, defined as $z_{ic} = \xi_{i,c}(Q)$ and $z_{is} = \xi_{i,s}(B)$, where the $\xi_{i,j}$ are the solution of the set of integral

equations [24]

$$\xi_{i,c}(k) = \delta_{i,c} + \int_{-B}^{B} \mathrm{d}\Lambda\ a_1(\Lambda - \sin k)\xi_{i,s}(\Lambda)$$
$$\xi_{i,s}(\Lambda) = \delta_{i,s} - \int_{-B}^{B} \mathrm{d}\Lambda'\ a_2(\Lambda - \Lambda')\xi_{i,s}(\Lambda') + \int_{-Q}^{Q} \mathrm{d}k\ \cos k a_1(\Lambda - \sin k)\xi_{i,c}(k). \tag{4.5}$$

The dressed charges simplify in the zero-field limit, i.e. when $B \to \infty$, where $z_{cs} = 0$, $z_{ss} = 2^{-1/2}$, and $z_{sc} = z_{cc}/2$, so the matrix of dressed charges then just depends on one non-universal parameter, z_{cc}. The dependence of the group velocities and z_{cc} on U and the band-filling is discussed in references [1, 6].

The AB and AC phase shifts for the Hubbard model are given by [10, 11, 16]

$$\vartheta_c = \frac{\phi}{\phi_0} + \frac{F}{F_0} \qquad \vartheta_s = \frac{2F}{F_0} \tag{4.6}$$

since up-spin electrons carry charge and spin, and spinons reduce the magnetization by $\overline{h}$. Here $\phi_0 = hc/e$ is the magnetic flux quantum and $F_0 = hc/\mu$ is its electric analogue, where μ is the magnetic moment of the electron. For the AB effect we have $F = 0$ and the temperature dependence of the persistent current is

$$j_c(T;\phi) = 4\pi T \sum_{l=c,s} z_{cl} \sum_{n=1}^{\infty} (-1)^n \frac{\sin\big[2\pi n(\phi/\phi_0 + D_c)\big]}{\sinh(\pi n L T/v_l z_{cl})} \tag{4.7}$$

where the parity of the oscillations depends on whether N_s is even or odd, i.e. each spin flip changes the parity. As a function of temperature, the ($T = 0$) saw-tooth [10, 16] is gradually smeared (the amplitude decreases dramatically) and becomes more sinusoidal as the higher harmonic content is suppressed.

The AC effect ($\phi = 0$), on the other hand, shows two periodicities as a function of F, namely F_0 and $F_0/2$. The temperature dependence of the persistent spin current is

$$j_s(T;F) = 4\pi T \sum_{n=1}^{\infty} (-1)^n \sum_{l=c,s} \frac{z_{cl} + 2z_{sl}}{\sinh(\pi n L T/v_l z_{cl})} \sin\big[2\pi n(F/F_0 + D_c)\big]$$
$$+ 4\pi T \sum_{n=1}^{\infty} (-1)^n \sum_{l=c,s} \frac{z_{cl} + 2z_{sl}}{\sinh(\pi n L T/v_l z_{sl})} \sin\big[2\pi n(2F/F_0 + D_s)\big] \tag{4.8}$$

where the parity, i.e. whether D_c and D_s take integer or half-integer values, again depends on the number of charges N_c and reversed spins N_s. As a function of temperature the amplitude of the higher-harmonic content is strongly reduced. The AC effect at $T = 0$ has been studied in references [10, 16].

4.2. The supersymmetric t–J model

The t–J model is defined by the Hamiltonian

$$H = -\sum_{i\sigma} P\big(c^{\dagger}_{i\sigma}c_{i+1\sigma} + c^{\dagger}_{i+1\sigma}c_{i\sigma}\big)P + J\sum_{i\sigma\sigma'}\Big[c^{\dagger}_{i\sigma}\boldsymbol{S}_{\sigma\sigma'}c_{i\sigma'} \cdot c^{\dagger}_{i+1\sigma'}\boldsymbol{S}_{\sigma'\sigma}c_{i+1\sigma} - \frac{1}{4}n_{i\sigma}n_{i\sigma'}\Big] \tag{4.9}$$

where $c^{\dagger}_{i\sigma}$ creates an electron of spin σ at site i, $n_{i\sigma} = c^{\dagger}_{i\sigma}c_{i\sigma}$, the $\boldsymbol{S}$ are spin-1/2 operators, and P is a projector that excludes the double occupancy of the sites. The model is integrable at the supersymmetric point $J = 2$ (see references [1, 27, 28]), where we equated the hopping to 1. The supersymmetric t–J model has been diagonalized within the framework

of Bethe's *ansatz* in terms of two sets of quantum numbers, $\{I_n\}$, $n = 1, \ldots, N_u$ for N_u unpaired electrons carrying charge and spin, and $\{J_\alpha\}$, $\alpha = 1, \ldots, N_p$ for N_p pairs of electrons (spin singlets). All quantum numbers within one set have to be different (Fermi statistics). Again, these quantum numbers are integers or half-integers depending on the parities of N_u and N_p, i.e. $I_n = N_p/2$ (mod 1) and $J_\alpha = (N_u + N_p + 1)/2$ (mod 1). In the ground state the quantum numbers are densely distributed, forming a sequence from I_{min} to I_{max} and from J_{min} to J_{max}, respectively, without leaving holes, so the backward-scattering quantum numbers are $2D_u = I_{min} + I_{max}$ (mod 1) and $2D_p = J_{min} + J_{max}$ (mod 1).

The occupation of states for the ground state is governed by the dressed energy potentials, $\varepsilon(k)$ and $\psi(\Lambda)$, for the unpaired and paired states, respectively, satisfying the Fredholm integral equations [28]

$$\begin{aligned}\varepsilon(p) &= -2 + 2\pi a_1(p) - \mu - H/2 - \int_{|\Lambda|>Q} \mathrm{d}\Lambda\ a_1(\Lambda - p)\psi(\Lambda) \\ \psi(\Lambda) &= -4 + 2\pi a_2(\Lambda) - 2\mu - \int_{|\Lambda'|>Q} \mathrm{d}\Lambda'\ a_2(\Lambda - \Lambda')\psi(\Lambda') - \int_{|p|>B} \mathrm{d}p\ a_1(\Lambda - p)\varepsilon(p)\end{aligned} \tag{4.10}$$

where now $a_n(\Lambda) = (n/2\pi)/(\Lambda^2 + (n/2)^2)$. The Fermi surfaces and the integration limits are determined by the zeros of the energies, $\varepsilon(\pm B) = 0$ and $\psi(\pm Q) = 0$. The distribution densities for the rapidities, $\rho(p)$ and $\sigma'(\Lambda)$, satisfy similar integral equations [1, 28]:

$$\begin{aligned}\rho(p) &= a_1(p) - \int_{|\Lambda|>Q} \mathrm{d}\Lambda\ a_1(\Lambda - p)\sigma'(\Lambda) \\ \sigma'(\Lambda) &= a_2(\Lambda) - \int_{|\Lambda'|>Q} \mathrm{d}\Lambda'\ a_2(\Lambda - \Lambda')\sigma'(\Lambda') - \int_{|p|>B} \mathrm{d}p\ a_1(\Lambda - p)\rho(p).\end{aligned} \tag{4.11}$$

The elementary excitations of the t–J chain are given by $\Delta E_u(p) = |\varepsilon(p)|$ and $\Delta E_p(\Lambda) = |\psi(\Lambda)|$ with respective momenta

$$p_u(p) = 2\pi \int_{-\infty}^{p} \mathrm{d}p'\ \rho(p') \qquad p_p(\Lambda) = 2\pi \int_{-\infty}^{\Lambda} \mathrm{d}\Lambda'\ \sigma'(\Lambda').$$

The excitation energies vanish at the Fermi points and the group velocities are

$$v_u = -\left.\frac{\partial\varepsilon}{\partial p}\right|_B [2\pi\rho(B)]^{-1} \qquad v_p = -\left.\frac{\partial\psi}{\partial\Lambda}\right|_Q [2\pi\sigma'(Q)]^{-1}. \tag{4.12}$$

The matrix of dressed generalized charges is defined as $z_{iu} = \xi_{i,u}(B)$ and $z_{ip} = \xi_{i,p}(Q)$, where the $\xi_{i,j}$ are the solutions of [15]

$$\begin{aligned}\xi_{i,u}(p) &= \delta_{i,u} - \int_{-B}^{B} \mathrm{d}p'\ a_2(p - p')\xi_{i,u}(p') + \int_{-Q}^{Q} \mathrm{d}\Lambda\ a_1(\Lambda - p)\xi_{i,p}(\Lambda) \\ \xi_{i,p}(\Lambda) &= \delta_{i,p} + \int_{-B}^{B} \mathrm{d}p\ a_1(\Lambda - p)\xi_{i,u}(p).\end{aligned} \tag{4.13}$$

In zero field ($B \to \infty$), the dressed charges are $z_{pu} = 0$, $z_{uu} = 2^{-1/2}$, and $z_{up} = z_{pp}/2$, so the matrix of dressed charges then just depends on one non-universal parameter, z_{pp}. The dependence of the group velocities and z_{pp} on the band filling is discussed in references [1, 15].

For the supersymmetric t–J model the two rapidity bands correspond to unpaired (simple charge and spin) and paired (double-charged singlet) electrons, so the phase shifts are

$$\vartheta_u = \frac{\phi}{\phi_0} + \frac{F}{F_0} \qquad \vartheta_p = \frac{2\phi}{\phi_0}. \tag{4.14}$$

The situation is then reversed with respect to the Hubbard chain, namely for the t–J model the AB effect is the superposition of two periods and the AC effect has a single period. For the AB effect ($F = 0$) the temperature dependence of the persistent current is given by

$$j_c(T;\phi) = 4\pi T \sum_{n=1}^{\infty}(-1)^n \sum_{l=u,p} \frac{z_{ul} + 2z_{pl}}{\sinh(\pi n LT/v_l z_{ul})} \sin\bigl[2\pi n(\phi/\phi_0 + D_u)\bigr] + 4\pi T \sum_{n=1}^{\infty}(-1)^n \sum_{l=u,p} \frac{z_{ul} + 2z_{pl}}{\sinh(\pi n LT/v_l z_{pl})} \sin\bigl[2\pi n(2\phi/\phi_0 + D_p)\bigr] \qquad (4.15)$$

where the parity, i.e. whether D_u and D_p take integer or half-integer values, depends on the number of unpaired (N_u) and spin-paired (N_p) electrons. Again the higher-harmonic content is strongly reduced with temperature.

The AC effect ($\phi = 0$) at $T = 0$ is a simple saw-tooth of period F_0. The temperature dependence of the persistent spin current is then

$$j_s(T;F) = 4\pi T \sum_{l=u,p} z_{ul} \sum_{n=1}^{\infty}(-1)^n \frac{\sin\bigl[2\pi n(F/F_0 + D_u)\bigr]}{\sinh(\pi n LT/v_l z_{ul})} \qquad (4.16)$$

where the parity of the oscillations depends on whether N_p is even or odd, i.e. the addition of a pair changes the parity. As a function of temperature the ($T = 0$) saw-tooth is gradually smeared and becomes more sinusoidal as the higher-harmonic content is suppressed.

5. Conclusions

Charged particles couple to the magnetic field flux threading the ring, giving rise to an AB quantum interference pattern and a persistent charge current. The dual effect is the AC quantum interference of particles carrying a magnetic moment which couples to a radial electric field at the metallic ring. At $T = 0$ the persistent currents have a generalized saw-tooth shape consisting of piecewise straight segments. The parity of the oscillations is controlled by the total number of electrons and the magnetization. The amplitude of the oscillation is determined by the spacing of the energy levels in the ring, $\hbar v_F/L$, i.e. they arise from the mesoscopic corrections to the ground-state energy. The persistent currents are then determined by the Luttinger parameters of the system, i.e. the group velocities and the matrix of dressed generalized charges. The latter describes the coupling of the various Fermi points. We considered a correlated electron system, which, in view of the charge–spin separation in one dimension, has a low-energy excitation spectrum given by two Dirac seas (two classes of excitations).

The temperature changes the saw-tooth to a more sinusoidal pattern with the same periodicity but strongly reduced amplitude. T suppresses the higher-harmonic content, as a consequence of the smearing of the Fermi surface. The controlling dimensionless parameter is (LT/v_F), i.e. the thermal energy over the level spacing. The level spacing depends on the energy band, and the quantum number varied. The characteristic temperature distinguishing between the high- and low-T regimes is $T_0 = v_F/\pi L$. The physical picture is the same as in the de Haas–van Alphen effect, where T introduces incoherence in the wavefunction while a particle completes its closed orbit. Scattering off imperfections along the ring gives rise to a mean free path and an effective Dingle temperature. Consequently, the observed oscillations are not saw-tooth-like, but always nearly sinusoidal.

In section 4 we applied the general results to two integrable models, namely the Hubbard chain and the supersymmetric t–J model. Within the framework of Bethe's *ansatz* these models have different ground-state structure (charges and spinons versus unpaired and paired

electrons). These structures are believed to be representative for a large number of models and not specific to the integrability. The results presented in this paper extend calculations (using the bosonization of fermions [21] and the Bethe *ansatz* [23]) for systems with only one Fermi surface (spinless fermions with charge) to the more realistic situation of electrons with charge and spin. The interference of the Fermi surfaces in the present case may lead to more than one periodicity (period halving).

The results presented in this paper are expected to hold for all models with a Luttinger liquid representation for the low-energy excitations. This is satisfied for most one-dimensional systems with gapless excitation spectra. A model with the (linear) Luttinger liquid spectrum extended to all energies is integrable. Small deviations from the linearized excitation spectrum away from the Fermi surface may spoil the integrability, but without changing the relevant low-energy properties. Hence, the charge and spin stiffnesses are the same, independently of the integrability.

The authors of reference [29] studied a tagged spinless particle coupling to the flux that interacts with a lattice gas of spinless fermions. This model can be interpreted as a two-band model with only one of the bands having a Fermi surface. The low-energy excitations are therefore not of the Luttinger liquid type, except when the hoppings in the two bands are equal. In this case the model has an additional symmetry and is integrable. The fact that the stiffness for this model is nonzero at finite temperature only in the integrable case is therefore not in contradiction to the results presented here. The model can alternatively be considered as an impurity model with a non-Fermi-liquid low-temperature fixed point [30, 31].

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